

CIE A-LEVEL  
Topical Past Papers

# **PURE MATHEMATICS 3**


## **9709**

2017 — 2025


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# CHAPTER 1


## Algebra

**1 - (9709/31\_Summer\_2017\_Q1)** 


Solve the inequality  $|2x + 1| < 3|x - 2|$ . [4]

**2 - (9709/31\_Summer\_2017\_Q2)** 

Expand  $\frac{1}{\sqrt[3]{1 + 6x}}$  in ascending powers of  $x$ , up to and including the term in  $x^3$ , simplifying the coefficients. [4]

**3 - (9709/32\_Summer\_2017\_Q2)** 

Solve the inequality  $|x - 3| < 3x - 4$ . [4]

**4 - (9709/32\_Summer\_2017\_Q8)** 


$$\text{Let } f(x) = \frac{5x^2 - 7x + 4}{(3x + 2)(x^2 + 5)}.$$

(i) Express  $f(x)$  in partial fractions. [5]

(ii) Hence obtain the expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [5]

**5 - (9709/33\_Summer\_2017\_Q2)** 

Expand  $(3 + 2x)^{-3}$  in ascending powers of  $x$  up to and including the term in  $x^2$ , simplifying the coefficients. [4]

**6 - (9709/31\_Winter\_2017\_Q1)** 

Find the quotient and remainder when  $x^4$  is divided by  $x^2 + 2x - 1$ . [3]

7 - (9709/32\_Winter\_2017\_Q8)

**ANSWER**

$$\text{Let } f(x) = \frac{8x^2 + 9x + 8}{(1-x)(2x+3)^2}.$$

- (i) Express  $f(x)$  in partial fractions. [5]
- (ii) Hence obtain the expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [5]

8 - (9709/31\_Summer\_2018\_Q4)

**ANSWER**

The polynomial  $x^4 + 2x^3 + ax + b$ , where  $a$  and  $b$  are constants, is divisible by  $x^2 - x + 1$ . Find the values of  $a$  and  $b$ . [5]

9 - (9709/31\_Summer\_2018\_Q9)

**ANSWER**

$$\text{Let } f(x) = \frac{12x^2 + 4x - 1}{(x-1)(3x+2)}.$$

- (i) Express  $f(x)$  in partial fractions. [5]
- (ii) Hence obtain the expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [5]

10 - (9709/32\_Summer\_2018\_Q1)

**ANSWER**

Showing all necessary working, solve the equation  $3|2^x - 1| = 2^x$ , giving your answers correct to 3 significant figures. [4]

11 - (9709/32\_Summer\_2018\_Q9)

**ANSWER**

$$\text{Let } f(x) = \frac{x - 4x^2}{(3-x)(2+x^2)}.$$

- (i) Express  $f(x)$  in the form  $\frac{A}{3-x} + \frac{Bx+C}{2+x^2}$ . [4]
- (ii) Hence obtain the expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^3$ . [5]

## 1 - (9709/31\_Summer\_2017\_Q1)



<b>EITHER:</b> State or imply non-modular inequality $(2x + 1)^2 < (3(x - 2))^2$ , or corresponding quadratic equation, or pair of linear equations $(2x + 1) = \pm 3(x - 2)$	
Make reasonable solution attempt at a 3-term quadratic e.g. $5x^2 - 40x + 35 = 0$ or solve two linear equations for $x$	
Obtain critical values $x = 1$ and $x = 7$	
State final answer $x < 1$ and $x > 7$	
<b>OR:</b> Obtain critical value $x = 7$ from a graphical method, or by inspection, or by solving a linear equation or inequality	
Obtain critical value $x = 1$ similarly	
State final answer $x < 1$ and $x > 7$	
<b>Total:</b>	<b>4</b>

## 2 - (9709/31\_Summer\_2017\_Q2)



<b>EITHER:</b> State a correct unsimplified version of the $x$ or $x^2$ or $x^3$ term in the expansion of $(1 + 6x)^{-\frac{1}{3}}$	
State correct first two terms $1 - 2x$	
Obtain term $8x^2$	
Obtain term $-\frac{112}{3}x^3$ $\left(37\frac{1}{3}x^3\right)$ in final answer	
<b>OR:</b> Differentiate expression and evaluate $f(0)$ and $f'(0)$ , where $f'(x) = k(1 + 6x)^{-\frac{4}{3}}$	
Obtain correct first two terms $1 - 2x$	
Obtain term $8x^2$	
Obtain term $-\frac{112}{3}x^3$ in final answer	
<b>Total:</b>	<b>4</b>

## 3 - (9709/32\_Summer\_2017\_Q2)



<i>EITHER:</i> State or imply non-modular inequality $(x-3)^2 < (3x-4)^2$ , or corresponding equation	
Make reasonable attempt at solving a three term quadratic	
Obtain critical value $x = \frac{7}{4}$	
State final answer $x > \frac{7}{4}$ only	
<i>OR1:</i> State the relevant critical inequality $3-x < 3x-4$ , or corresponding equation	
Solve for $x$	
Obtain critical value $x = \frac{7}{4}$	
State final answer $x > \frac{7}{4}$ only	
<i>OR2:</i> Make recognizable sketches of $y =  x-3 $ and $y = 3x-4$ on a single diagram	
Find $x$ -coordinate of the intersection	
Obtain $x = \frac{7}{4}$	
State final answer $x > \frac{7}{4}$ only	
<b>Total:</b>	<b>4</b>

## 4 - (9709/32\_Summer\_2017\_Q8)



(i)	State or imply the form $\frac{A}{3x+2} + \frac{Bx+C}{x^2+5}$	
	Use a relevant method to determine a constant	
	Obtain one of the values $A = 2, B = 1, C = -3$	
	Obtain a second value	
	Obtain the third value	
	<b>Total:</b>	<b>5</b>

(ii)	Use correct method to find the first two terms of the expansion of $(3x + 2)^{-1}$ , $(1 + \frac{3}{2}x)^{-1}$ , $(5 + x^2)^{-1}$ or $(1 + \frac{1}{5}x^2)^{-1}$ [Symbolic coefficients, e.g. $\binom{-1}{2}$ are not sufficient]	
	Obtain correct unsimplified expansions up to the term in $x^2$ of each partial fraction. The FT is on A, B, C. from part (i)	
	Multiply out up to the term in $x^2$ by $Bx + C$ , where $BC \neq 0$	
	Obtain <b>final answer</b> $\frac{2}{5} - \frac{13}{10}x + \frac{237}{100}x^2$ , or equivalent	
	<b>Total:</b>	<b>5</b>

## 5 - (9709/3. \_Summer\_2017\_Q2)



<p><i>EITHER:</i></p> <p>State a correct unsimplified version of the <math>x</math> or <math>x^2</math> term in the expansion of <math>(1 + \frac{2}{3}x)^{-3}</math> or <math>(3 + 2x)^{-3}</math></p> <p>[Symbolic binomial coefficients, e.g. <math>\binom{-3}{2}</math>, are not sufficient for <b>M1</b>.]</p>	
State correct first term $\frac{1}{27}$	
Obtain term $-\frac{2}{27}x$	
Obtain term $\frac{8}{81}x^2$	
<p><i>OR:</i></p> <p>Differentiate expression and evaluate <math>f(0)</math> and <math>f'(0)</math>, where <math>f'(x) = k(3 + 2x)^{-4}</math></p>	
State correct first term $\frac{1}{27}$	
Obtain term $-\frac{2}{27}x$	
Obtain term $\frac{8}{81}x^2$	
<b>Total:</b>	<b>4</b>

## 6 - (9709/31\_Winter\_2017\_Q1)



Commence division and reach a partial quotient $x^2 + kx$	
Obtain quotient $x^2 - 2x + 5$	
Obtain remainder $-12x + 5$	
	<b>3</b>

## 7 - (9709/32\_Winter\_2017\_Q8)



(i)	State or imply the form $\frac{A}{1-x} + \frac{B}{2x+3} + \frac{C}{(2x+3)^2}$	
	Use a relevant method to determine a constant	
	Obtain one of the values $A = 1, B = -2, C = 5$	
	Obtain a second value	
	Obtain the third value	
		<b>5</b>
	[Mark the form $\frac{A}{1-x} + \frac{Dx+E}{(2x+3)^2}$ , where $A = 1, D = -4, E = -1$ , <b>B1M1A1A1A1</b> as above.]	
(ii)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$ , $(1+\frac{2}{3}x)^{-1}$ , $(2x+3)^{-1}$ , $(1+\frac{2}{3}x)^{-2}$ or $(2x+3)^{-2}$	
	Obtain correct unsimplified expansions up to the term in $x^2$ of each partial fraction	
	Obtain final answer $\frac{8}{9} + \frac{19}{27}x + \frac{13}{9}x^2$ , or equivalent	
		<b>5</b>

## 8 - (9709/31\_Summer\_2018\_Q4



<i>EITHER:</i> Commence division by $x^2 - x + 1$ and reach a partial quotient of the form $x^2 + kx$	
Obtain quotient $x^2 + 3x + 2$	
<i>Either</i> Set remainder identically equal to zero and solve for $a$ or for $b$ , or multiply given divisor and found quotient and obtain $a$ or $b$	
Obtain $a = 1$	
Obtain $b = 2$	
<i>OR:</i> Assume an unknown factor $x^2 + Bx + C$ and obtain an equation in $B$ and/or $C$	
Obtain $B = 3$ and $A = 2$	
<i>Either</i> Use equations to obtain $a$ or $b$ or multiply given divisor and found factor to obtain $a$ or $b$	
Obtain $a = 1$	
Obtain $b = 2$	
	<b>5</b>

## 9 - (9709/31\_Summer\_2018\_Q9



(i)	State or imply the form $A + \frac{B}{x-1} + \frac{C}{3x+2}$	
	State or obtain $A = 4$	
	Use a correct method to obtain a constant	
	Obtain one of $B = 3, C = -1$	
	Obtain the other value	
		<b>5</b>
(ii)	Use correct method to find the first two terms of the expansion of $(x-1)^{-1}$ or $(3x+2)^{-1}$ , or equivalent	
	Obtain correct unsimplified expansions up to the term in $x^2$ of each partial fraction	
	Add the value of $A$ to the sum of the expansions	
	Obtain final answer $\frac{1}{2} - \frac{9}{4}x - \frac{33}{8}x^2$	
		<b>5</b>

## 10 - (9709/32\_Summer\_2018\_Q1



<i>EITHER:</i>	State or imply non-modular equation $3^2(2^x - 1)^2 = (2^x)^2$ , or pair of equations $3(2^x - 1) = \pm 2^x$	
	Obtain $2^x = \frac{3}{2}$ and $2^x = \frac{3}{4}$ or equivalent	
<i>OR:</i>	Obtain $2^x = \frac{3}{2}$ by solving an equation	
	Obtain $2^x = \frac{3}{4}$ by solving an equation	
	Use correct method for solving an equation of the form $2^x = a$ , where $a > 0$	
	Obtain <b>final</b> answers $x = 0.585$ and $x = -0.415$ only	
		<b>4</b>

## 11 - (9709/32\_Summer\_2018\_Q9



(i)	Use a correct method to find a constant	
	Obtain one of the values $A = -3$ , $B = 1$ , $C = 2$	
	Obtain a second value	
	Obtain the third value	
		<b>4</b>
(ii)	Use a correct method to find the first two terms of the expansion of $(3-x)^{-1}$ , $\left(1 - \frac{1}{3}x\right)^{-1}$ , $(2+x^2)^{-1}$ or $\left(1 + \frac{1}{2}x^2\right)^{-1}$	
	Obtain correct unsimplified expansions up to the term in $x^3$ of each partial fraction	
	Multiply out their expansion, up to the terms in $x^3$ , by $Bx + C$ , where $BC \neq 0$	
	Obtain <b>final</b> answer $\frac{1}{6}x - \frac{11}{18}x^2 - \frac{31}{108}x^3$ , or equivalent	
		<b>5</b>